PREDRAG STOJKOV *Research*

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I have interests in areas of modern quantum field theory and nonlinear dynamical systems. My research and interests up to this point are specifically: functional methods in quantum field theory, nonlinear dynamical systems, application of Weyl-Moyal's approach to quantization of statistical and quantum systems, deformations of continuous symmetries, the Batalin-Vilkovitsky quantization. They are described in more details in following sections. I am looking forward to the study of the other areas in high-energy physics, in particular modern development in strings and supersymmetry, subjects which I have not had as yet had opportunity to study in detail.

A) Functional methods in quantum field theory

It is an annoying fact of field theorists' life that one can solve exactly only the most trivial, toy models. For more interesting models, even for the simplest such as ϕ^4 or QED, it is not known how to exactly evaluate the generating functional ("path") integral without resorting to a perturbative expansion or some kind of heuristic, semiclassical evaluation.

Based on methods initiated and developed by Schwinger and Fradkin, school of physicists has developed that is trying to perform exactly as many steps in the evaluation of the generating functional (and related quantities) as possible, before resorting to approximations. By insisting on such an approach, it is possible to a get set of approximate non-perturbative results, as in the case of eikonal (UV) like approximations.

For class of simple models (which includes ϕ^4 and gauge theories) it is possible to express the generating functional as formal, exact series of several, consecutive functional operations. These involve performing Gaussian linkage operations (equivalent to path integrals with Gaussian measures), a useful representation of exact Green's functions $G_c[A]$ in fixed (but arbitrary) external fields, and a knowledge of the corresponding loop generating functionals $L[A] \sim \det[G_c[A]]$. The exact representations of such a Green's functions are known (Fradkin, Fried & Gabellini) but their evaluation is still mainly an unsolved task.

My current research is centered about the methods to evaluate and approximate these exact representations of Green's functions $G_c(x, y|A)$ in arbitrary external fields. Based on the recent non-perturbative, eikonal-inspired hierarchy of approximations to $G_c[A]$ of Fried & Gabellini, I have devised an independent counterpart to their hierarchy of approximations. Most basic of those new approximations, the $< 0 >$-approximation (where the propagating field is behaving as extremely UV), is still giving exact results for cases of constant external field A, as well as for monochromatic plane waves (lasers).

Applied to the ϕ^4 model, the $< 0 >$-approximation gives an intriguing result in $1+3$ dimensions: the quantized model is free (noninteracting) and massless. In other numbers of dimensions, the conclusion is different, as in the case of $1 + 1$ where both mass and interaction survive quantization. Higher approximations (that are gradual improvements of $< 0 >$-approximation), preserve the free&massless result in $1 + 3$ dimensions, and it is tempting to consider this as the simplest analytical proof of triviality of the ϕ^4 model.

Another closely-related direction of my research is devising a hierarchy of approximations of the exact Green's functions, when the external field is gauge (as in QCD) or gravitational. Both cases involve additional technical difficulties due to non-commutativity, conveniently focused in (path) Ordered Exponentials (OEs). It is possible to map such matrix OEs into vertex operators, and to extract some additional information from them, due to better knowledge of how to approximately evaluate such operators.

One natural check of the above approximations is found in the Schwinger-DeWitt asymptotic expansion, which was successfully applied twenty-five years ago to the case of gravity (propagation in a curved background). For the case of a scalar background, both methods (functional approximation and Schwinger-DeWitt asymptotic expansion) are giving same results, reaffirming each other.

One of the goals of my present research is a comparison of the effective quantum behavior of particles and fields in presence of certain backgrounds with their classical motion in those fields. In particular, I have concentrated on laser-like external fields (in cases of scalar, electromagnetic and gravitational nature), finding new classical solutions. One line of research is considering the effects of gravitational waves on an electromagnetic laser: I have found the exact classical solution for that problem.

Although the development of the abovementioned hierarchy of approximations to $G_c[A]$ was just a one small logical step in exploring possibilities of functional approach, I have found it more valuable than expected. Presently, I am exploring properties and possibilities for application of the first few members of that hierarchy. One important application is the evaluation of functional determinants in external fields, and among them the approximate (valid in the long-wave domain) bosonization in $1 + d$ ($d \geq 1$) dimensions (in the spirit of recent research by Schaposnik on bosonization in $d = 2$).

I do hope to bring this research to a logical conclusion, in all the above aspects. The first paper on these methods is submitted to Physical Review D (and is accessible from Lanl bulletin board under the name hep-th/9706135), and subsequent two papers are in the preparation. A major part of this research has been motivated by my adviser H.M.Fried.

B) Non-linear Dynamical Systems

I am also interested in non-linear dynamical systems, in particular in extreme situations of exact solubility and of chaos.

Under the guidance and with the collaboration of M. Belić, I have performed numerical and elementary symbolical research in chaos (two of my published papers are in that field), and on recognizing hidden symmetry properties of certain chaotic systems (non-linear optical systems) and the role of those symmetries in their's partial solubility (other two publications). These two research projects were performed before I came to Brown, but they initiated my general interest into the interplay of ordered and chaotic aspects of complex phenomena.

Here at Brown I have taken several courses on nonlinear dynamical systems (mainly on Math and Applied Math departments), in order to increase my knowledge on mathematical aspects of complex systems.

Many interesting problems exists on the quantization of classically chaotic systems: I am exploring the control mechanisms that a quantum aspect imposes over classically chaotic systems. Research is developing along two lines: Firstly, exploiting exact Green's functions in external fields (that research is guided by my adviser), using the same forms explained in the previous Section; and secondly, exploring effective classical dynamical systems resulting from suitable truncations of full (infinite) quantum systems of ODE's (along the lines recently initiated by Sadov, observing the dynamical equations for the hierarchy of higher momenta of phase space operators). This second line of research is performed in colaboration with P. Nešković. The report on that research is in preparation.

C) Applications of Weyl-Moyal approach to quantization on statistical and chaotic systems

Another research project that I have performed at Brown (under my own direction) is in the Weyl-Moyal's approach to quantization. This interest is the offspring of a third possible approach to quantum chaos, a initiated recently by W.H. Zurek. His merit is to insist on a description of the quantum system (in terms of the Weyl-Moyal prescription) as the dynamics of Wigner's distribution function in classical phase space of the system, where dynamics is defined by Moyal's bracket, instead of its classical limit, Poisson bracket. This description allowed him to gain an extremely intuitive picture on role of competition between quantum diffusion and tendency towards forming homoclinic tangle present in classically chaotic system. This makes possible a simple, analytic expressions for a time scale when the behavior of a quantum system breaks away from its classical counterpart (such a behavior is visible in numerical simulations). I am interested in a possible extension of such an intuitive picture to the case of infinitely many degrees of freedom (fields), where additional aspect of spatially chaotic motion (turbulence) appears.

While learning Moyal's formalism, I have found some simple but intriguing results in its application to statistical systems. For example, for simplest systems that can be reduced to a set of harmonic oscillators, the quantum partition function is equivalent to a classical partition function for a classical system of oscillators with different temperature and nonzero chemical potential ("quantum temperature" and "quantum chemical potential"). These results can be extended in one direction to the case of Grassmann oscillators, and in other direction towards infinite many degrees of freedom (to noninteracting fields and to simple strings). A paper describing these results is in preparation.

D) Deformations of Continuous Symmetries

Finally, I should mention a long-term, research interest in deformed symmetries: qualitative preservation (while changing the explicit form of generators and structure functions) of continuous symmetries upon deformations (continuous changes from the identity) of underlying objects. Although the slightest such deformation violates the initial symmetry of system, the deformed form of symmetry stays in the system. What is really violated is the rigid, linear representation of the initial symmetry. If the deformation of the underlying object is very special, part of the linear realization/representation of symmetry will survive, and we usually talk of the breaking of a larger symmetry G to the smaller one H.

It turns out that, at least in the case of non-dynamical systems (such as algebraic hyper-surfaces and statistical Hamiltonians): 1) The deformed algebra always (for any value of deformational parameter) has an equal number of generators as does initial undeformed algebra. 2) In the case of an algebraic hyper-surface, if the topology of the underlying object (that hyper-surface) is not changed, then the deformed algebra (symmetry algebra of the deformed object) is isomorphic to the algebra of the initial, undeformed object. 3) Explicit expressions for deformed generators exist. 4) Part of the initial symmetry that is considered broken in the standard terminology can be represented in a nonlinear fashion. A paper on these results is in preparation.

In the application of these results to statistical Hamiltonians, some interesting points are worth mentioning: If the initial Hamiltonian H_0 has a certain continuous symmetry (with the algebra g_0), then on adding an additional term tH_1 the symmetry algebra of new Hamiltonian is s deformation g_t of g_0; but, as in case of coupled spin chains, the majority (all that are not linear symmetries of total Hamiltonian) of deformed generators become nonlinear and (in the continuous limit) nonlocal.

Also, in attempts to evaluate the statistical integral (path integral) for deformed system, an interesting point is that while symmetry remains present in Hamiltonian (figuring in the Boltzman factor of statistical integral), the naive phase space measure is non-invariant exactly upon the appearance of nonlinear generators. That is a point that which I still do not understand entirely: how is it possible to have a conspiracy of the quantum/statistical measure (anomaly) to discriminate out exactly the deformed piece of classical symmetry.

My goal is to understand equivalent phenomena arising in case of dynamical systems (ODEs, PDEs, classical field theories), and their change upon quantization. Results that I have obtained for such systems are still not transparent, and are not in the form suitable for publication.

E) Batalin-Vilkovitsky quantization

I should also mention some work accomplished before coming to Brown on BV quantization of constrained (superparticle) systems. In the case of the simplest superparticle in $D = 1 + 9$ dimensions, the Brink-Casalbuoni-Schwarz superparticle, the symmetry algebra is of infinite degree of reducibility. The naive application of the Batalin-Vilkovitsky procedure fails to produce a gauge-fixed pre-quantum action; residual symmetries are of Schtücklberg type. Under guidance of B. Sazdović and M. Blagojević I investigated the origin of the failure of Batalin-Vilkovitsky procedure in the BCS superparticle case, as well as a possible cure for this problem. These results were never published.